Period 1, Jan 27, 2025

$$\int (\tan^2 y + 1)\, dy$$

$$\tan^2 y + 1 = \frac{\sin^2 y}{\cos^2 y} + \frac{\cos^2 y}{\cos^2 y} = \frac{\sin^2 y + \cos^2 y}{\cos^2 y} = \frac{1}{\cos^2 y} = \sec^2 y$$

$$\int \sec^2 y\, dy$$

$$\tan y + C$$

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43. $$\int \frac{\cos x}{1-\cos^2 x}\, dx$$

$$\frac{\cos x}{\sin^2 x + \cancel{\cos^2 x} - \cancel{\cos^2 x}} = \frac{\cos x}{\sin^2 x} = \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$\cot x \cdot \csc x$$

$$\int \cot x \cdot \csc x\, dx$$

$$-\csc x + C$$

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44. $$\int \frac{\sin x}{1-\sin^2 x}\, dx$$

$$\frac{\sin x}{\cancel{\sin^2 x} + \cos^2 x - \cancel{\sin^2 x}} = \frac{\sin x}{\cos^2 x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\tan x \cdot \sec x$$

$$\int \tan x \sec x\, dx$$

$$\sec x + C$$

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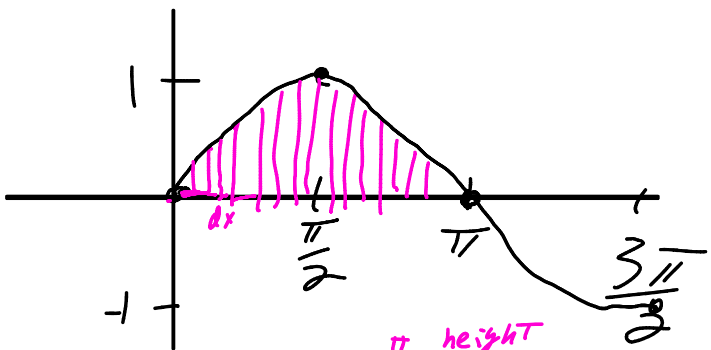
33. $$\int dx = \int 1\, dx = \int x^0\, dx = \frac{1}{1} x^{0+1=1} + C = x + C$$

$$\int \frac{1}{x\sqrt{x}}\, dx = \int \frac{1}{x^1 \cdot x^{\frac{1}{2}}}\, dx = \int \frac{1}{x^{3/2}}\, dx = \int x^{-3/2}\, dx = 1 \cdot -2 \cdot x^{-\frac{3}{2}+1=-\frac{1}{2}} + C$$

$$-2x^{-\frac{1}{2}} + C = \frac{-2\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}} + C$$

$$\frac{-2\sqrt{x}}{x} + C$$



$\frac{\pi-0}{16}$ = Riemann sum with $n=16$

$\lim_{n\to\infty} \frac{\pi}{n} = dx$

$\int_0^\pi \sin x \cdot dx$ height width

$\int_0^\pi \sin x \, dx$

$-\cos x + c \Big[_0^\pi$

$[-\cos\pi + c] - [-\cos 0 + c]$

$-\cos\pi + \cancel{c} + \cos 0 - \cancel{c}$

$-(-1) + 1 = 1 + 1 = 2$

Example Set 1: Sketch the region corresponding to each definite integral. Then evaluate each integral using a geometric formula.

$\int 4dx = 4x + c$

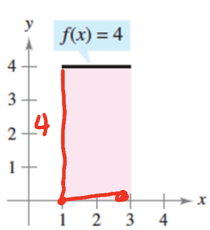
$f(x) = 4$

$$\int_1^3 4dx = 8$$

$4(3) - 4(1)$

$12 - 4 = 8$

Geometrical shape? Rectangle!



$2 \cdot 4 = 8$

$A = bh$

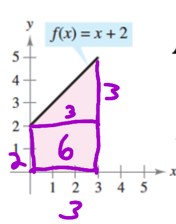
$A = (3-1)(4)$

$A = 8$

$f(x) = x + 2$

$$\int_0^3 (x+2)dx = \frac{21}{2}$$

Geometrical shape? Trapezoid!



$A = \frac{1}{2}(b_1 + b_2)h$

$A = \frac{1}{2}(2+5)3$

$= \frac{21}{2}$

$\frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2}$

$\frac{9}{2} + 6 = 10\frac{1}{2} = \frac{21}{2}$

$\frac{1}{2}x^2 + 2x + c \Big[_0^3$

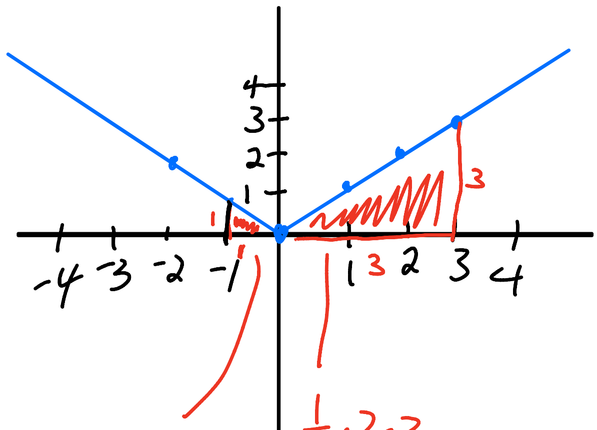
$\frac{1}{2}(3)^2 + 2(3) - [\cancel{\frac{1}{2}(0)^2 + 2(0)}]$

0

$\frac{9}{2} + 6$

$\frac{9}{2} + \frac{12}{2} = \frac{21}{2} = 10\frac{1}{2}$

Student Example 1: Evaluate the integral $\int_{-1}^{3} |x|\,dx = 5$



$\frac{1}{2}\cdot 1\cdot 1 + \frac{1}{2}\cdot 3\cdot 3$

$\frac{1}{2} + \frac{9}{2} = \frac{10}{2} = 5$

x	$y=\lvert x\rvert$
0	0
1	1
-1	1
2	2
-2	2
3	3
-3	3

$\int_{-1}^{3} |x|\,dx = 5$

$\int_{3}^{-1} |x|\,dx = -5$

$\int_{-1}^{-1} |x|\,dx = 0$

$\int_{-1}^{4} F(x)\,dx = 7$

$\int_{4}^{6} F(x)\,dx = 2$

$\int_{-1}^{6} F(x)\,dx = 7+2$

$\int_{-1}^{4} 4\,F(x)\,dx = 4\int_{-1}^{4} F(x)\,dx = 4\cdot 7 = 28$

Example 3: Consider the function $f(x) = 3 - x$. Sketch a graph of this function.

A) What is the area between the curve and the x-axis between x = 4 and x = 8?

$\frac{1}{2}(-1+-5)\cdot 4$

$\frac{1}{2}\cdot -6\cdot 4 = 2\cdot -6$

$= -12$

$\int(3-x)\,dx = 3x - \frac{1}{2}x^2 + C \Big[_4^8$

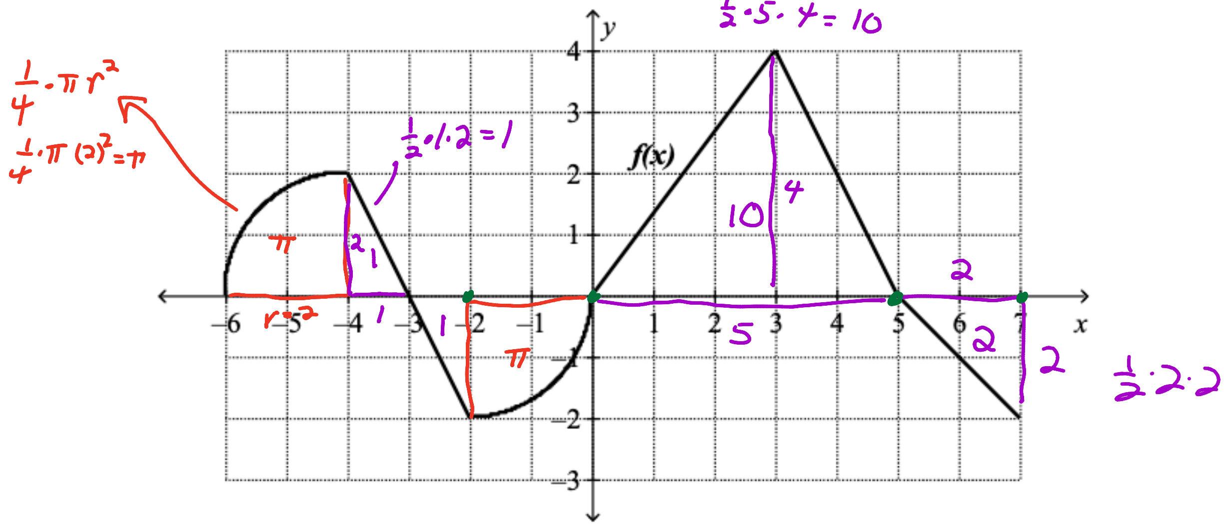
$3(8) - \frac{1}{2}(8)^2 - [3(4) - \frac{1}{2}(4)^2]$

b) $\int_4^8 f(x)\,dx$

$24 - \frac{1}{2}(64) - [12 - \frac{1}{2}(16)]$

$24 - 32 - 12 + 8 = \cancel{32} - \cancel{32} - 12$

Example Set 4: Use the graph above to find the indicated integrals.



a) $\int_0^5 f(x)dx = 10$ b) $\int_5^0 f(x)dx = -10$ c) $\int_{-2}^7 f(x)dx$

$\int_5^{-2} F(x)dx = -10+\pi$

$-\pi + 10 - 2 = 8-\pi$

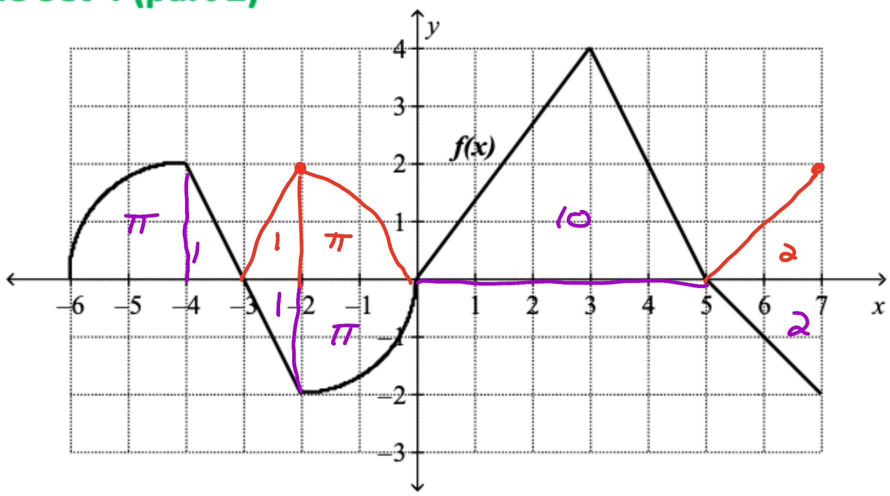
$\int_0^3 (x^2+2)dx = \frac{1}{3}x^3+2x+c \Big[_0^3 = \frac{1}{3}(3)^3 + 2(3) + c - \left[\cancel{\frac{1}{3}(0)^3+2(0)+c}\right]$

$= \frac{1}{3}(27)+6 = 9+6 = 15$

$\int_3^0 (x^2+2)dx = \frac{1}{3}x^3+2x+c \Big[_3^0 = \frac{1}{3}(0)^3+2(0)+c - \left[\frac{1}{3}(3)^3+2(3)+c\right]$

$= 0+0+\cancel{c} - 9 - 6 - \cancel{c} = -15$

Example Set 4 (part 2)



Use the graph above to find the indicated integrals.

d) $\int_{-6}^{5} f(x)dx$

$\cancel{\pi} + \cancel{1} - \cancel{1} - \cancel{\pi} + 10$

10

e) $\int_{-6}^{5} [f(x) + 2]\, dx = 10 + 22$

$\boxed{32}$

$\int_{-6}^{5} F(x)dx + \int_{-6}^{5} 2\,dx$ $\quad 10 + 22$

$10 + 2x + c \Big[_{-6}^{5}$

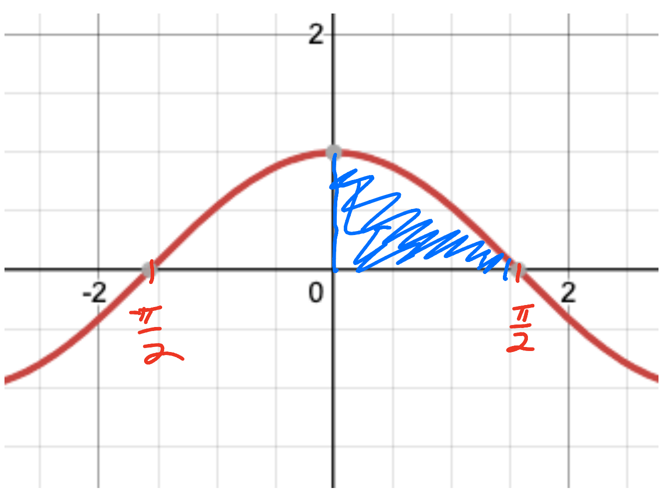
$2(5) + c - [2(-6) + c]$

$10 + c + 12 - c = 22$

f) $\int_{-6}^{7} |f(x)|\, dx$

$\pi + 1 + 1 + \pi + 10 + 2$

$2\pi + 14$



$y = \cos x$

$\int_{-\frac{\pi}{2}}^{\pi/2} \cos x\, dx = 2\int_{0}^{\pi/2} \cos x\, dx = 2$

$2\sin x \Big[_{0}^{\pi/2}$

$2\sin\frac{\pi}{2} + c - [2\sin 0 + c]$

$2 \cdot 1 + \cancel{c} - [2 \cdot 0 + \cancel{c}] = 2$

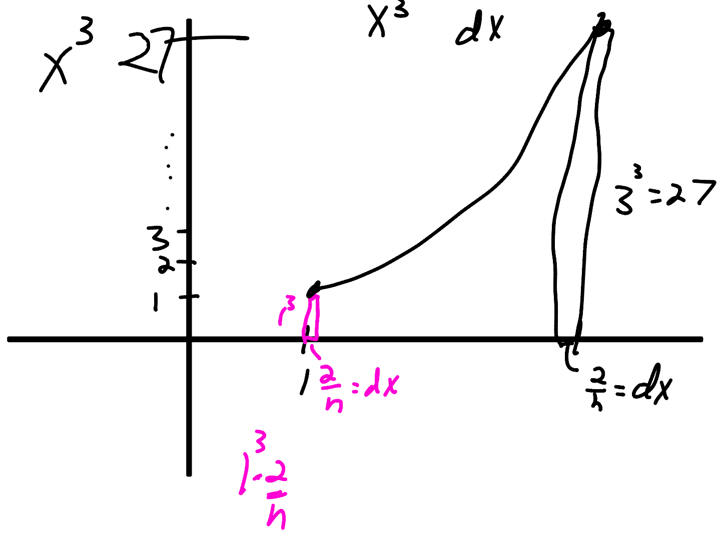
1) Write a **definite integral** equal to the following limit:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\underbrace{\left(1+\frac{2k}{n}\right)^3}_{x^3}\underbrace{\frac{2}{n}}_{dx} = \lim_{n\to\infty}\underbrace{\frac{2}{n}}_{\text{width}}\overbrace{\left[\left(1+\frac{2(1)}{n}\right)^3+\left(1+\frac{2(2)}{n}\right)^3+\left(1+\frac{2(3)}{n}\right)^3+\dots\right.}^{\text{heights}}$$

$(1+0)^3$

$$\left.\left(1+\frac{2(n-1)}{n}\right)^3+\left(1+\frac{2\cdot \cancel{n}}{\cancel{n}}\right)^3\right]$$

$(3)^3$



$$\int_1^3 x^3\,dx$$

2) Which of the following **limits** is equal to $\int_0^{1} \sqrt{1+2x}\,dx$? $\quad \sqrt{1} \to \sqrt{3}$

(A) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\sqrt{1+\frac{k}{n}}\cdot\frac{1}{n}$ $\quad \sqrt{1} \to \sqrt{2}$

(B) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\sqrt{1+\frac{2k}{n}}\cdot\frac{1}{n}$ $\quad \sqrt{1} \to \sqrt{3}$

(C) ~~$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\sqrt{1+\frac{k}{n}}\cdot\frac{2}{n}$~~ spread $=2$

(D) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\sqrt{\frac{2k}{n}}\cdot\frac{1}{n}$ $\quad \sqrt{0} \to \sqrt{2}$